

1. Consider the skew projection onto the plane $x - 2y + z = 0$ with vector $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$
 - a. Find the matrix of the skew projection
 - b. Calculate the skew projection of the vector $\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$.
 - c. Find the image of the plane $x - 2y + z = 3$ with respect to this projection (don't calculate if you know the answer without calculating).
2. Consider the reflection with respect to the plane $x + y - z = 0$.
 - a. Find the matrix of the reflection.
 - b. Calculate the reflection of the vector $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$.
 - c. Find the image of the line $\vec{x} = t \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ with respect to this transformation
 - d. Calculate the image of the plane $x - y + 2z = 2$ w.r.t. this plane.
3. Consider the rotation with respect to the y axis with angle 60° .
 - a. Write the matrix of the rotation.
 - b. Find the image of the vector $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ with respect to this rotation.
 - c. Find the image of the plane $x - y + z = 2$ with respect to this rotation.
4. Consider the rotation with respect to the vector $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ with 45° .
 - a. Write the matrix of the rotation.

- b. Find the image of the vector $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ with respect to this rotation.
5. Consider the linear transformation determined by the matrix $M = \frac{1}{3} \begin{bmatrix} 0 & 1 & -2 \\ -6 & 5 & -4 \\ -3 & 1 & 1 \end{bmatrix}$.
- Show that it is a skew projection.
 - Find the plane and skew vector $\vec{v}$ for this projection. (the plane P so that the matrix M is the reflection with respect to P).
6. Solve the eigenvalue eigenvector problem for $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$.
7. Solve the eigenvalue eigenvector problem for $A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.
8. Given the affine transformations $T(\vec{x}) = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $U(\vec{x}) = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \vec{x} + \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, find the compositions:
- $U \circ T(\vec{x})$.
 - $T \circ U(\vec{y})$.
9. Find the inverse of the affine transformation $T(\vec{x}) = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.
10. Consider the projection onto the line $2x + y = 3$.
- Write the projection as an affine transformation.
 - Calculate the projection of the point $A(3, -1)$.
11. Consider the reflection w.r.t. the line $2x - y = 3$.
- Write it as an affine transformation.
 - Calculate the reflection of the line $x + y = 1$ with respect to this line.
12. Consider the rotation with 30° around the point $(1, 2)$.
- Find the rotation as an affine transformation.
 - Find the rotation of the image of the line $x - y = 2$ under this rotation.
13. Consider the projection onto the plane $x + y - z = 2$.
- Write the projection as an affine transformation.

- b. Calculate the projection of the point $A(3, -1, 2)$.
14. Consider the reflection w.r.t. the plane $x - y + z = 2$.
- a. Write it as an affine transformation.
- b. Calculate the reflection of the plane $x + y + z = 3$ with respect to this plane.
15. Consider the affine transformation $T(\vec{x}) = \frac{1}{3} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \vec{x} + \frac{1}{3} \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$.
- a. Show that it is a projection.
- b. Find the plane of this projection.